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A study on the optimization method for a multi-body system using the response surface analysis †

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Abstract

An optimization method, which minimizes the characteristic value of a system using response surface analysis, is presented. Plackett-Burman design is used as a screening method. Using the response surface analysis, second order recursive model function is estimated as an objective function. To verify the reliability of the model function, an F-test based on the analysis of variances table is used. Lastly, the sequential quadratic-programming method is used to find the value of design parameters. By applying the preceding procedure to a multi-body dynamic model, the optimization process presented in this study is verified.

Keywords: Design of experiments; Plackett-Burman design; Central composite design; Analysis of variance; Sequential quadratic programming method

1. Introduction

As the production cycle of goods gets shorter, analysis and designs using a computer model become all the more important. A reliable simulation model reduces design time and improves efficiency by guiding the design direction of a real system. Recently, many numerical optimization methods have been developed and widely used to design better systems.

System optimization methods may be classified as analytical and experimental. Analytical methods derive an objective function from the equations of motion, which are comprised of many independent and dependent variables. Following this step, one can then solve the optimization problem. Although this method has good accuracy and reliability [1], it is not widely used because the equations of motion must be defined according to each system. In contrast, an experimental method performs the minimum number of experiments based on statistical theory and finds the relationship between design parameters and the response variable using results from experiments [2]. Thus, the objective function can be obtained easily without deriving the theoretical equations of motion of a system.

In this study, an optimal design method for a multi-body system using the response surface analysis is presented. First, in order to choose the design parameters, which significantly affect the response variable, sensitivity analysis is performed by Plackett-Burman (P-B) design table [3]. Then, experiments are made according to the central composite (C-C) design table [4]. Next, using response surface analysis [5], the second order recursive model function, which informs the relationship between design parameters and the response variable, is estimated. The reliability of the

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estimated model function is verified according to the analysis of variance (ANOVA) method. Finally, the sequential quadratic-programming (SQP) method [6] is used to find the value of design variables, which minimizes the model function and satisfies linear or nonlinear constraint conditions. To verify the reliability of the above optimization procedure, a multi-body simulation model of the wiper system is created. The displacement of the tip of the left and right blades of the target, current and optimized system is compared, and the usefulness of the optimal design method presented is verified.

2. Theoretical background

2.1 Optimization procedure

Fig. 1 shows the systematic optimization procedure using D.O.E. The first step is to define the characteristic value, design parameters and constraints. The characteristic value, called a response variable, is the output value of the system to be maximized or minimized. When defining design parameters, the level of parameters also needs to be defined. The level is the number of values that the parameters can take. For example, a 2-level parameter can take only a maximum and minimum value, whereas a 3-level parameter can take a minimum, neutral and maximum value. Constraints are conditions that design parameters must satisfy. Most constraints are linear or nonlinear algebraic equations. Next, a sensitivity analysis is performed to select the parameters that have a significant influence on the change of the characteristic value. After the design parameters are chosen, the relationship between the characteristic value and design parameters should be defined as a function. The response surface analysis uses the second order polynomial recursive model to estimate the function, assuming this function is nonlinear. To find the recursive model function using a small number of experiments, the central composite design table, an orthogonal array that adds the central and axis point to the 2-level factor experiment, is used. The model function is estimated by using the experiment results according to the least square method. The estimated model function is verified with an F test based on the ANOVA table. The verified model function is used as an objective function in the optimization problem. Finally, according to the SQP method, the optimum value of design parameters that minimizes the objective function and satisfies constraints is found.

2.2 Response surface analysis

The response surface analysis is a method used to estimate the relationship between the design parameters and the characteristic value. When design parameters $x_1, x_2, \ldots, x_n$ affect the characteristic value y, the change of the characteristic value according to these design parameters can be defined as the second order surface. Then this surface is made into a statistical model, called a response function. In this study, the second order polynomial recursive model function is used for the response function. The response function is determined by the experiment results according to the central composite design. The second order recursive model function consisting of k design parameters is defined as

$$y = b_0 + \sum_{i=1}^{k} b_i x_i + \sum_{j=1}^{k}\sum_{i=1}^{j} b_{ij} x_i x_j + e \tag{1}$$

where x is the level of design parameters according to each experiment condition, β is the coefficients of design parameters, y is the characteristic value,

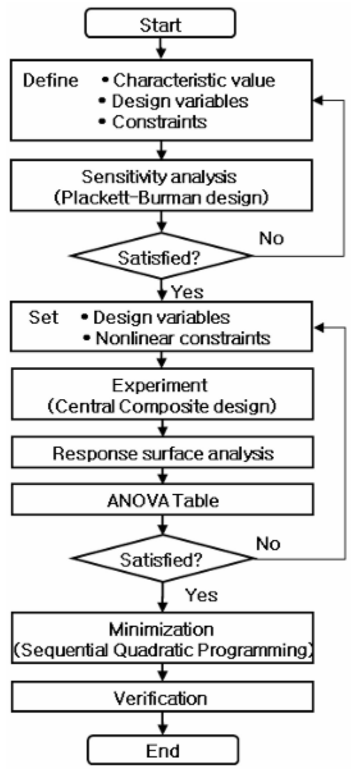

Fig. 1. Optimization procedure.

ε is the error and k is the number of design parameters. Eq.(1) is expressed in a matrix form as Eq.(2).

$$Y = XB + E \tag{2}$$

According to the least square method (Burden *et al*, 2001), recursive coefficients (β) can be calculated by minimizing the square of error and the form is

$$B = (X^T X)^{-1} X^T Y \tag{3}$$

To verify the estimated model function, the ANOVA table is made and an F test is conducted.

3. Optimization example

In this study, a wiper system as shown in Fig. 2 is chosen as an optimization target. When a wiper system is designed, the wiping area, which is the area of the windshield that a blade should scrub, is considered as the most important factor. The wiping area is measured according to the angle of eyes of a driver and passenger after the windshield and seat are designed. Then, blades and arms that cover the wiping area are designed, and the linkage system is created.

To show the optimization process presented in this paper is robust and reliable, the positions of the points 1 and 2 are optimized to make the displacement of the tips of the left and right blades follow the target curve. The length of blades and arms and the connecting angle between blades and arms are assumed to be constant. The y coordinates of two points are also assumed to be the same. Table 1 shows values of design variables. A sensitivity analysis was not performed since the number of design variables is just 3. When the number of design variables is up to 5, the whole design variables can be optimized at a time. If the number of design variables is greater than 5, however, the minimization algorithm may not find the global minimum point, and thus the optimization results are not reliable.

The objective of the optimization is to minimize the average difference between the target curve and the estimated curve of the tip of the right and left blades. Fifteen Experiments are run according to the central composite design as shown in Table 2, since there are three design variables whose level is three.

In Table 2, x_1, x_2 and x_3 are design variables, and y is the average difference. According to the response surface analysis method, the 2nd order recursive model function is derived as

$$\begin{aligned} y = 13.0194 + 8.3476x_1 - 4.0924x_2 + 1.0935x_3 \\ + 3.7938x_1^2 + 0.884x_2^2 + 1.7168x_3^2 \\ - 5.1868x_1x_2 + 0.0646x_2x_3 - 0.104x_1x_3 \end{aligned} \tag{4}$$

To verify the reliability of the estimated recursive model function, an F-test using the ANOVA table is performed as shown in Table 3. In the table, F_0 is greater than F(0.01). Therefore, the estimated model

Table 1. Design variables.

Design variables		Level		
		-1	0	1
Point 1	X coord. (x_1)	-40	-35	-30
	Y coord. (x_2)	2	5	8
Point 2	X coord. (x_3)	-35	-30	-25

Table 2. Experiment table and results.

No	x_1	x_2	x_3	y
1	-1	-1	-1	8.0592
2	-1	-1	1	9.9999
3	-1	1	-1	13.2998
4	-1	1	1	15.8573
5	1	-1	-1	34.0254
6	1	-1	1	35.9086
7	1	1	-1	18.8773
8	1	1	1	20.6601
9	0	0	0	14.4346
10	-1.216	0	0	6.1522
11	1.216	0	0	30.1757
12	0	-1.216	0	24.3645
13	0	1.216	0	3.3583
14	0	0	-1.216	13.5233
15	0	0	1.216	16.6624

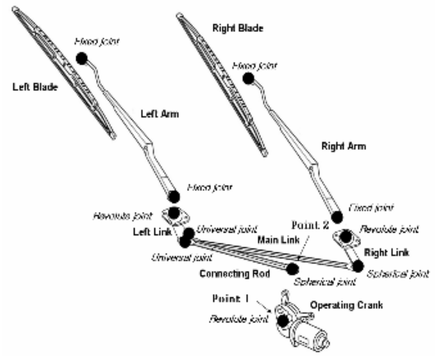

Fig. 2. Multi-body model of a wiper system.

Table 3. Analysis of variable (ANOVA) table.

Factor	S	ϕ	V	F_0	F(0.01)
Reg.V	1254.6	3	418.2	47.46	6.22
Res.V	96.9	11	8.8		
Total	1351.5	14			

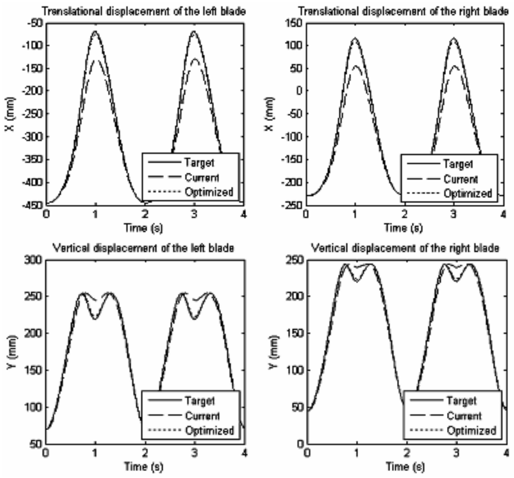

Fig. 3. Multi-body model of a wiper system.

function is considered to be reliable at a significance level of 1%, and the model function can be used as the objective function. The objective function, design variables, and the constraint conditions are then all defined. The value of the design variables that minimize the objective function is by found using the SQP method.

Fig. 3 shows the translational and vertical displacement of the tip of the left and right blade. In this figure, the solid line is the target displacement, the dashed line is the current displacement, and the dotted line is the optimized displacement. As shown in the figure, the optimized curve is much closer to the target curve than the current curve. Therefore, the optimization is performed successfully and the optimization procedure presented in this paper is verified.
v

4. Conclusions

An optimization procedure using the design of experiments is presented in this paper. The objective function that shows the relationship between design variables and the response of the system is estimated according to the response surface analysis method, and verified by F-test using the ANOVA table. To verify the robustness of the optimization procedure, the displacements of the tips of the left and right blades of the wiper system are optimized. Simulation results show that the optimized displacement follows the target displacement well, and thus the optimization procedure using the response surface analysis is verified.

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References

[1] J. S. Arora, *Introduction to Optimum Design,* McGraw-Hill, New York, (1994).
[2] C. R. Hicks, *Fundamental Concepts in the Design of Experiments*, Holt, Rinehart and Winston Inc, New York, (1973).
[3] R. Plackett and J. Burman, The Design of Optimum Multifactorial Experiments. *Biometrica,* 33 (1946) 305-325.
[4] S. H. Park, *Understanding of Design of Experiments*, Minyoungsa, Seoul, (2006).
[5] R. H. Myers, *Response Surface Methodology*, Allyn & Bacon, Inc., Boston, (1971).
[6] N. Vanderplaats, *Numerical Optimization Techniques for Engineering Design with Applications*, McGraw-Hill, New York, (1984).

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